# The Cops and Robbers on Circulant Di-Graphs

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# Cops and Robbers Rules

## Rules

- At the start of the game:
  1. Cops are placed on the vertices of $G$
  2. The Robber chooses a vertex of $G$
- During the game:
  - Cops and the Robber alternate moving to adjacent vertices
- Win conditions:
  - Cops win if one cop can move to the vertex occupied by the Robber
  - The Robber wins if they can avoid the cops indefinitely

# Cops and Robbers Rules

## Definition

The primary objective in the study of cops and robbers is to determine the minimum number of cops required to apprehend a robber on a given graph. This minimum value is known as the cop number of the graph and is denoted as $c(G)$ for graph $G$. Simply finding a strategy to catch the robber with a given number of cops is insufficient; the goal is to demonstrate that fewer cops cannot successfully capture the robber. Achieving this often involves identifying both upper and lower bounds such that the upper bound is equivalent to the lower bound.

# Previous Findings

- Finding by Shannon L. Fitzpatrick and John Paul Larkin (2017) ( On Undirected Circulant Graphs)
- $c(G) = 2$ if and only if $G \cong \text{Circ}(n; m)$ for any $n \geq 4$, or $G \cong \text{Circ}(n; m, k)$ where $n \geq 6$ and $n$, $m$, $k$ satisfy at least one of the following:
  1. $k = 2m$
  2. $k = 3m$
  3. $n = 2k$
  4. $n = 2k + 2m$
  5. $n = 3k$, and $4m = 3k$ or $3m = 2k$
  6. $n = 3m$, and $4m = 3k$

# Special Case with $C(G) = 2$

**Lemma**

- If $n \geq 4$ and $k = 2$, then $C(Circ(n; 1, k)) = 2$.

**Lemma**

- If $n \geq 4$ and $k = n - 1$, then $C(Circ(n; 1, k)) = 2$.

**Lemma**

- If $n \geq 4$ and $k = \frac{n}{2}$, then $C(Circ(n; 1, k)) = 2$.

# Special Case with $C(G) = 2$

**Lemma**

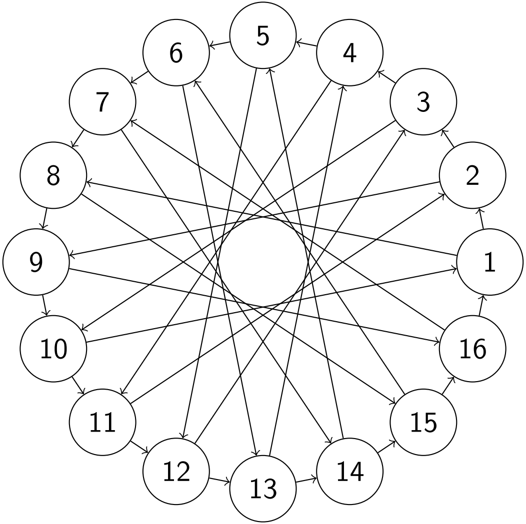
- If $n \geq 4$ and $k = n - 2$, then $C(Circ(n; 1, k)) = 2$

# Major Findings

**Lemma : Sandwich Lemma**

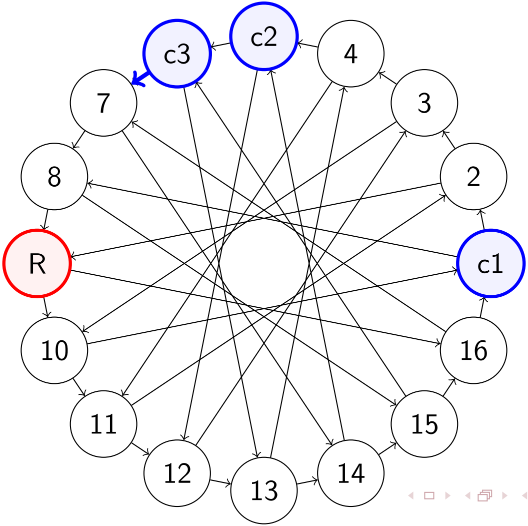
- If $n > 6$ and either $5 \leq k < \frac{n}{2}$ or $\frac{n}{2} < k \leq n - 5$, then there exists a two-step strategy to always capture the robber.

# example of sandwich lemma

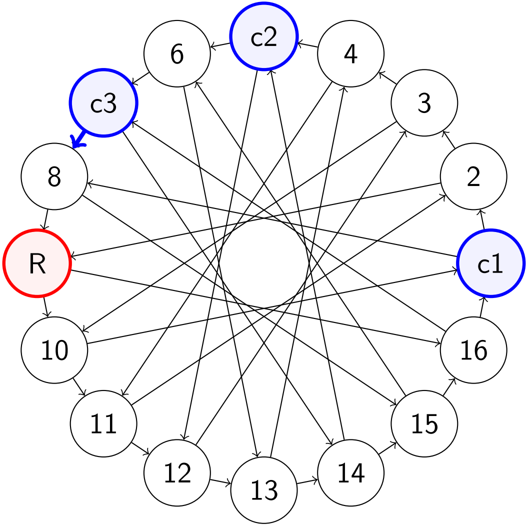


# example of sandwich lemma

- 1st Step



# example of sandwich lemma



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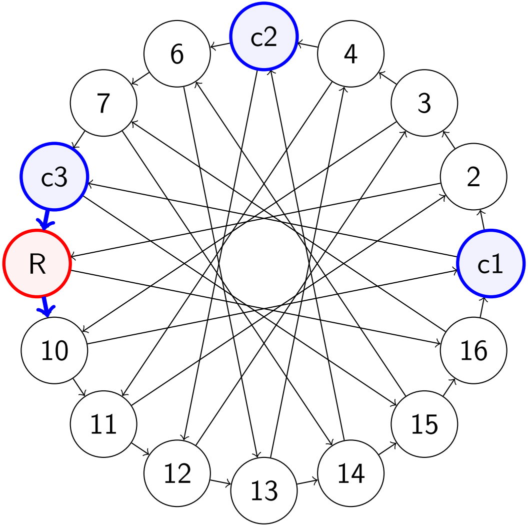

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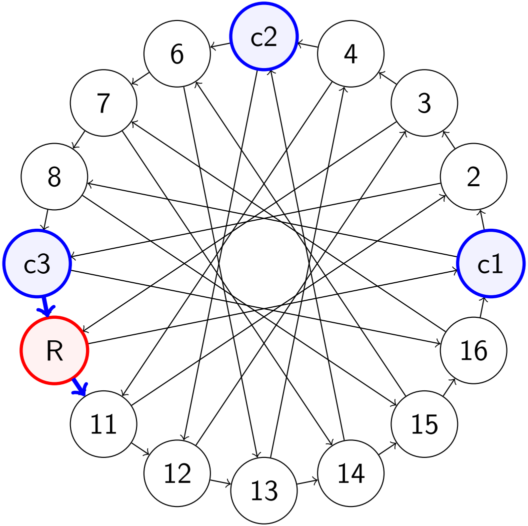
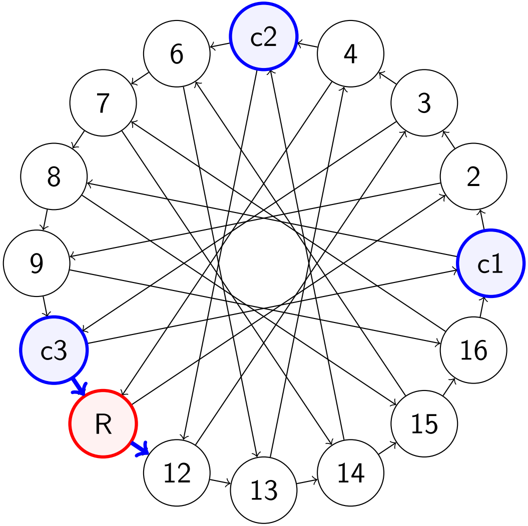
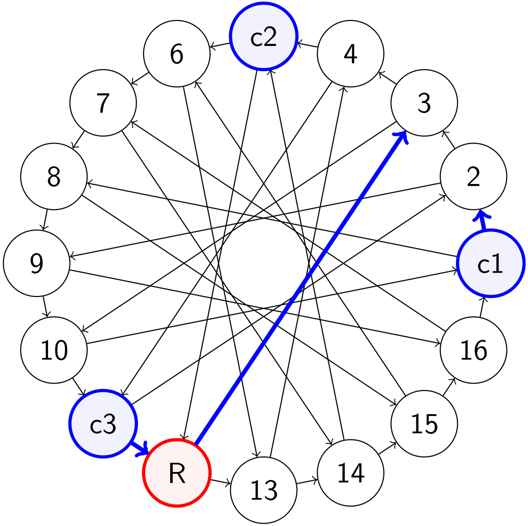


# example of sandwich lemma

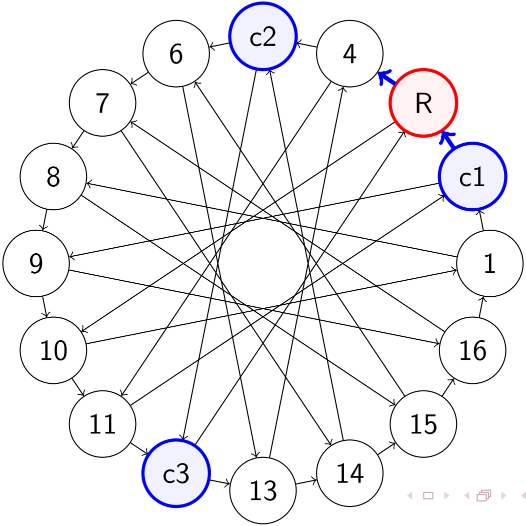


# example of sandwich lemma

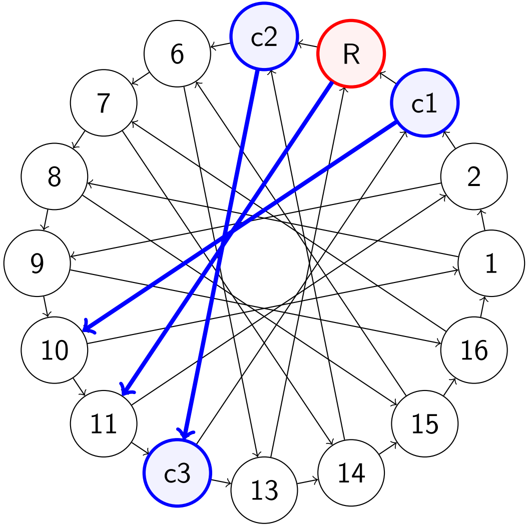


# example of sandwich lemma

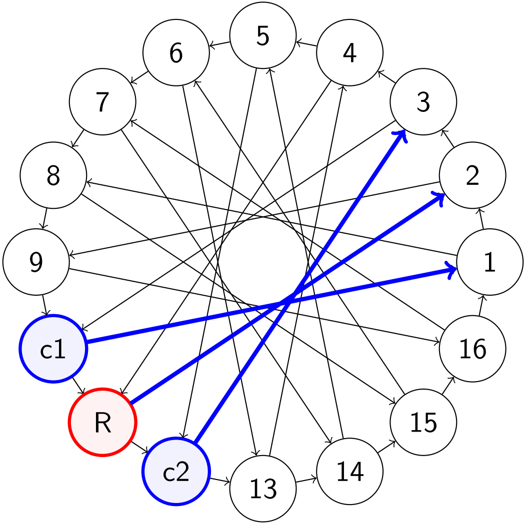
- 2nd Step



# example of sandwich lemma



# example of sandwich lemma



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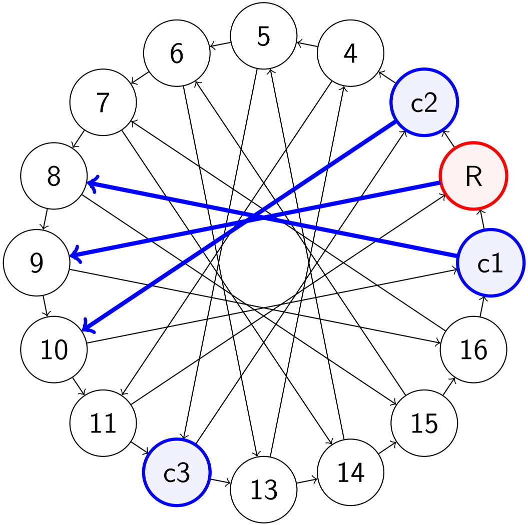
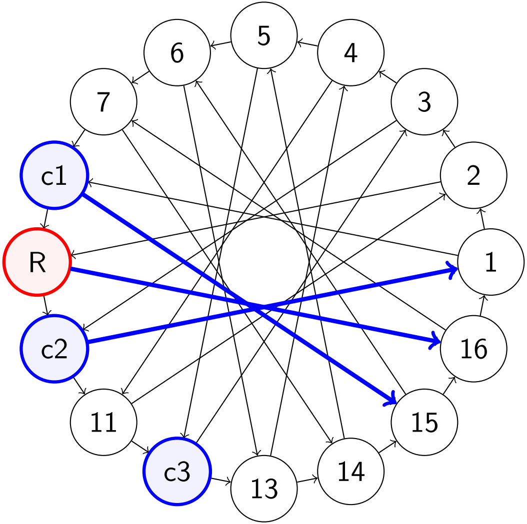


# example of sandwich lemma



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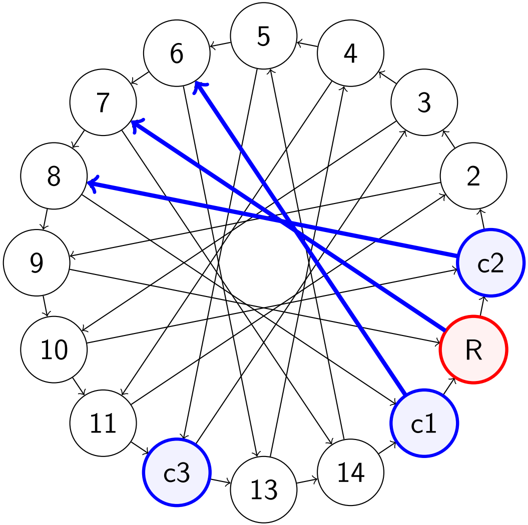

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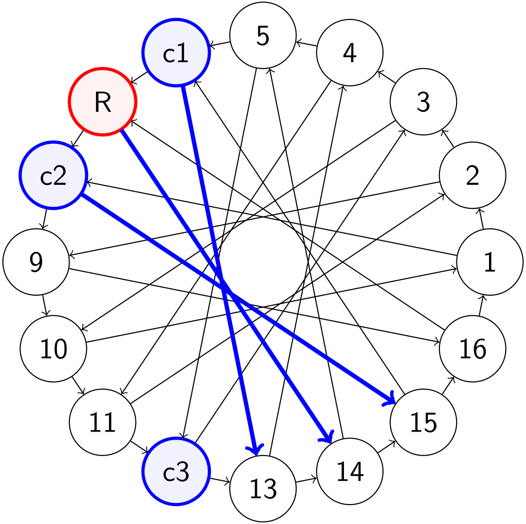

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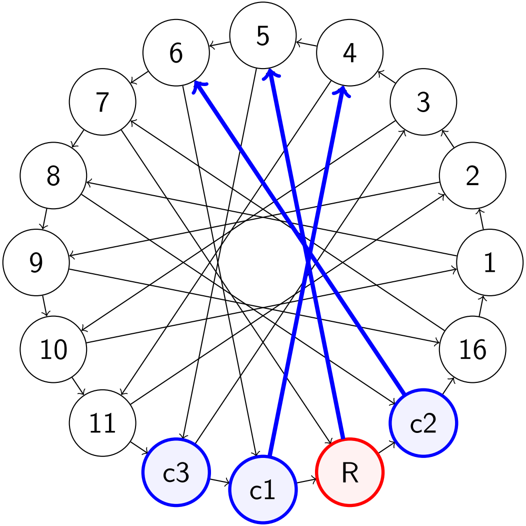

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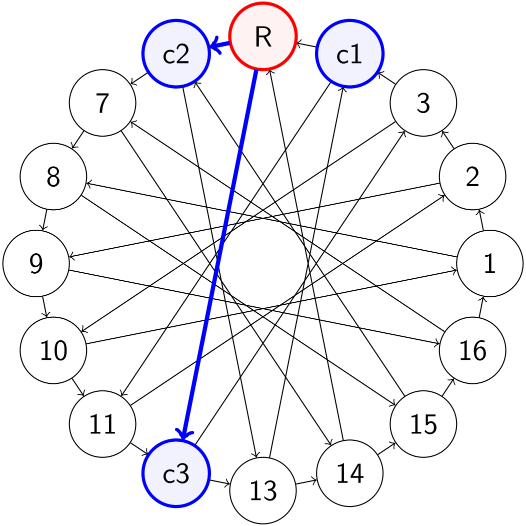


- Hence it is a Capture and $C(Circ) = 3$

# Generalized Case Lemma

**Lemma**

- If $n > 6$ and $3 \leq k < \frac{n}{2}$ or $\frac{n}{2} < k \leq n - 3$, then $C(Circ(n; 1, k)) > 2$

# Lemma 7: Final Lemma

**Lemma**

- If $n > 6$ and $3 \leq k < \frac{n}{2}$ or $\frac{n}{2} < k \leq n-3$, then $C(Circ(n;1,k)) = 3$

# Theorem and Conclusion

**Theorem**

- The number of cops required to capture the robber on a circulant graph $Circ(n;1,k)$ is given by:

$$C(Circ(n;1,k)) = \begin{cases} 2, & \text{if } n \text{ is even, } k = \frac{n}{2} \\ 2, & \text{if } k = 2 \\ 2, & \text{if } k = n-2 \\ 2, & \text{if } k = n-1 \\ 3, & \text{otherwise.} \end{cases}$$

# Theorem and Conclusion

**proof idea**

The proof proceeds by considering different cases based on the values of $n$ and $k$. For each case, specific lemmas are applied to determine the cop number for the circulant graphs with certain properties. In Case 1, we utilize Lemma for $k = \frac{n}{2}$, while in Cases 2 to 4, we employ Lemma's for particular values of $k$. For all other cases in Case 5, we apply the Generalized Lemma to establish the cop number for circulant graphs with $n > 6$ and specific ranges of $k$. By examining all possible cases, it is concluded that for $n \geq 4$ and $k \geq 2$, the cop number for the circulant graph $Circ(n; 1, k)$ is either 2 or 3, depending on the specific values of $n$ and $k$ as stated in the theorem.

# Fututre directions

**Lemma**

1. Finding the Cop Number of $Circ(n; i, j)$
2. Finding the Cop Number of $Circ(n; i, j, k, ...)$

# Questions? Comments?

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https://bit.ly/2XGQeZ9

THANKS!